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Review of solid mechanics in tribology

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Abstract

The study of solid mechanics is essential to the field of tribology, (friction, lubrication and wear). Tribology is of immense economic importance. The potential savings, were tribological principles better understood and applied to friction and wear reduction) may be several percent of the gross national product. Solutions to tribology problems often enable current technologies in a broad spectrum of applications from friction contact in the turbine shrouds of aircraft engines, to bearing contact in motor vehicle gear assemblies, to the sliding contact of magnetic storage disk drives. Conversely, tribology issues, e.g., the coefficient of friction, may impact solid mechanics problems and tangential tractions are essentially free parameters in many cases.

Active issues of research in tribology where solid mechanics is applied include: friction and wear in dynamic loading of bearings to extend bearing life; models for contact and thermal stresses of sliding surface asperities; design criteria for magnetic recording heads, and behavior of human artificial joints to extend service life.

Countless other applications exist, requiring the development of essential theories of conforming and non-conforming surface behavior. Information such as the frictional response of surfaces in relative motion, and modes of stress and deformation emerges from the fusion of solid mechanics and tribology. © 1999 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Tribology is the science and technology of interacting surfaces in relative motion. The word itself was first used in England the 1960's and comes from the Greek work 'tribos' meaning 'to rub'. The term was coined as a conscious attempt to combine the historically independent fields of friction, lubrication and

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wear in an interdisciplinary manner, as well as to attach a scientific sounding name to studies which were, for the most part, at that time, very applied. The attempt seems to have succeeded and the term tribology has found wide acceptance in both science and engineering. The study of fluid film bearings, rolling element bearings, seals, gears, cams, viscous dampers, human joints, and magnetic storage devices are some of the applications in which tribology is currently used.

Tribology is of immense economic importance, which certainly motivates its study. An improvement in the engineering practice of tribology through better understanding of a contact mechanics problem may involve savings in the billions of dollars. The Jost Report Jost (1966), in which the term tribology was first used, attributed about £515M/y (about 1% of the British gross national product) of potential savings were tribological principles better understood and applied. Such a huge economic impact was much derided at the time as self-serving exaggeration; however, the economics may have been greatly understated. The Jost Report primarily focused on energy loss due to friction but overlooked the much large pervasive costs of wear on maintenance, loss due to breakdowns, depreciation of machinery, etc. Such potential savings through tribology are huge, but clearly very difficult to rigorously quantify. Recent textbooks discuss the economic impact of tribology in their introductions, Rabinowicz (1995), and Hutchings (1992).

The history of the subject dates back to the studies of friction by Thermistius in 350 BC who found that the friction for sliding is greater than that for rolling. This finding led to the understanding in modern terms that the static friction coefficient is greater than the kinetic coefficient of friction. First noted in the 1500's by da Vinci, re-discovered by Amontons in 1699, verified by Euler in 1750 and Coulomb in 1781: each found that friction is proportional to load and independent of the area of sliding surfaces. Thus the coefficient of friction is independent of load, and in the case of dry (unlubricated) sliding, independent of velocity. Dowson (1997) presents an entertaining history of the field.

Little fundamental understanding into solid mechanics aspects of tribology was gleaned until this century when measurements could be taken of surface roughness, and inferences made as to the real area of contact between surfaces. Even the smoothest surfaces are rough on the atomic scale and contact only occurs at the tips of asperity peaks, Bowden and Tabor (1967). At first the deformation is elastic in the manner of the Hertz problem between spherical surfaces, see the discussion below. For metal surfaces, eventually the elastic limit is exceeded and plastic deformation occurs. With fully plastic asperity deformation, the real area of contact A_r is the (global) normal force P divided by the yield hardness H, $A_r = P/H$, and the friction force Q for dry sliding is the real area of contact times the shear strength Y. The asperity junctions are assumed to be joined by adhesion and then ruptured during sliding. Thus the friction coefficient is shear strength divided by hardness, $\mu = Q/P = Y/H$.

With similarly simplistic reasoning, a dimensionless wear coefficient K can be defined, Archard (1953), which is wear volume divided by real contact area times sliding distance, $K = Vol/A_rS = Vol\ H/PS$. If the plastically deformed zone below the asperity is the same order as the real contact area, then K represents a ratio of worn volume to the plastically deformed zone. For adhesive wear, K loosely represents the probability that an asperity adhesive junction leads to a wear particle. Adhesive wear occurs when asperities are in contact accompanied by high local pressures and sometimes the resulting weld can be stronger than the bulk asperity; the cohesive strength of the softer material being less than the interfacial strength. For abrasive wear, where the asperity material is harder than the material surface through which it is ploughing, a simple ploughing idealization leads to $K = \tan \vartheta/\pi$ where ϑ is the cutting angle, Rabinowicz (1995). For adhesive wear K is order 10^{-4} to 10^{-3}, and for abrasive wear K is order 10^{-1}.

In recent times, tribology is often a so-called pacing technology being crucial to a wide range of applications including high temperature engines made of ceramics, machine tools, metal cutting and forming processes, and biotechnology to name a few. Whether counter surfaces are conforming or non-conforming, made of like or unlike materials, lightly or highly loaded, under steady or dynamic loads, the study of solid mechanics is integral in solving tribological problems.

Many of the present day tribology problems, just as during its early beginnings, require knowledge of a combination of several areas of science. Researchers in the field come from a wide variety of backgrounds: surface science, chemistry of lubricants, machine design and behavior, material science, rheology, fluid mechanics and the like. Relatively few of the past and presently well-known people in tribology come from the solid mechanics field. Of the three areas of friction, wear and lubrication, only the latter has been broadly tractable to a theoretical foundation until very recently. In 1886, Osborne Reynolds developed his theory of hydrodynamic lubrication and the equation that bears his name. Reynolds' equation is a solution to the governing equations of Newtonian fluid mechanics (Navier–Stokes) for a thin confined film in which a three-dimensional nonlinear equation can be integrated to a two-dimensional linear partial differential equation. Solutions to Reynolds' equation form the basis of design and analysis of fluid film bearings. Probably due to the relatively straightforward nature of Reynolds' equation, many researchers have approached tribology from a fluid mechanics perspective.

The field of solid mechanics as applied to contacting surfaces began in the same era as lubrication theory with the publication of Heinrich Hertz's classic paper 'On the Contact of Elastic Solids' (Hertz, 1882). As pointed out by K.L. Johnson in the preface to his text *Contact Mechanics* (Johnson, 1987), Hertz' theory was confined to the case of frictionless surfaces and perfectly elastic solids. Removal of the former restriction has led to more realistic consideration of the sliding and rolling contacts of machine elements. Much early work in this direction is due to R.D. Mindlin (1949). Development of theories of plasticity and viscoelasticity has allowed application of solid mechanics to a wider range of materials and conditions. However, by contrast with lubrication, no single governing equation or set of equations could be said to adequately define the solid mechanics of friction and wear problems.

Most applications of solid mechanics to tribology are concerned with non-conformal surfaces, which touch first at a point or along a line. Even under finite load, the region affected by the contact is much smaller than the dimensions of the bodies themselves. The contact zone can generally be regarded as a region of stress concentration within the larger body. Strictly speaking, by most conventions, these types of problems comprise the field of contact mechanics. Most of this paper concerns contact mechanics in the sense just described, but we will use the term to mean 'solid mechanics aspects of tribology.'

By contrast, for conformal surfaces, stress is distributed over a large region and the contact problem cannot be separated from the overall stress distribution. The classical example of the non-conformal surface interaction is rolling contact such as between a bearing ball and race, with or without lubrication. On these problems, studies abound, as discussed below. The classical example of the conformal surface is the journal bearing - lubricant trapped between two concentric cylinders of nearly the same diameter. Solid mechanics studies of conformal surfaces are more rare, such as that of Kumar et al. (1990), concerning the elastic elongation of connecting rod bearings and housings during the engine loading cycle.

Most of the classical problems of the contact mechanics field have been attacked by building up stress distributions as an integral superposition of point force or line force solutions, i.e., the Green's function convolution integral approach. This is the primary approach of Johnson's text, *Contact Mechanics* (Johnson, 1987), which very adequately reflects the state of the field as of a decade ago. An earlier book *Surface Mechanics* by F.F. Ling (1973) seems to have been the first to have specifically addressed solid mechanics aspects of surface and interface phenomena. A wider range of more formal and sophisticated mathematical techniques is explored in the book *Contact Problems in the Mathematical Theory of Elasticity* by Gladwell (1980).

The purpose of many tribological studies is to predict friction and wear. Global friction and wear are due to a summation of the effect of many asperity interactions, each of which may be idealized as a micro-contact mechanics problem. Much recent solid mechanics-based tribological research has to do with models for rough surface contact, one of the earliest being due to Greenwood and Williamson (1966). They assume that for nominally flat surfaces the summit height distribution is Gaussian with height variance σ, and each summit can be replaced with a parabola of a specified radius of curvature R. From this model one can compute load P and real contact area A_r as a function of h/σ, where h is the mean surface separation. In addition a quantity known as the plasticity index $\psi = \sqrt{\sigma/R}(E'/H)$ may be computed which determines the onset of plastic deformation of the asperities. There are numerous roughness models along this line, and many studies that make use of such models.

Although the three aspects of tribology, those of friction, lubrication and wear, in most cases are interrelated, in the context of solid mechanics the areas of attention are primarily those of friction and wear. The results of most solid mechanics analyses are stresses and displacements and the relation of these quantities to the global variables of tribological interest, e.g., friction and wear, may not be clear. Experimentation remains the primary mode for determining such quantities as rates of wear and coefficients of friction. Material deformations and associated stresses play important, but largely uncertain, roles in determining the quantities of friction and wear. However, until such time that a means to mathematically model friction coefficients or wear rates from first principles exists, the experimental approaches will trump mathematical analysis in the study of tribological processes.

Tribology is a varied and diffuse topic. The remainder of this paper is organized around technologies on which current research on tribology is conducted, noting where solid mechanics issues arise, and then around tribology research issues. In the latter case, certain areas are avoided, notably, the huge effort toward nano-tribology and atomistic approaches both modeling and instrumentation. The interface of continuum and atomistic views of solid mechanics is discussed elsewhere in this volume. Furthermore, a more focussed review of contact mechanics *per se* may also be found in these pages.

2. Technologies

With tribology comprising numerous aspects, its study is approached from the direction of many disciplines with solid mechanics woven throughout. For this reason it is not easy to break out solid or contact mechanics as a consistent, isolated topic within tribology. Several tribology-driven technologies are highlighted below. The technologies selected and the associated papers discussed are not intended to be comprehensive but rather to illustrate some aspects of the field and certain issues therein. The selection below provides a mix new developments (magnetic storage and joint prostheses) and the traditional (gears and seals).

2.1. Hard disk drives

Digital technology is pushing the boundaries of magnetic disk recording with an ever-decreasing gap between the recording head and disk. Improvements in storage density and signal resolution are realized when the space between the magnetic medium and magnetic head is very small. The tribological difficulties arise from the nanoscale clearance between these moving surfaces. During normal operation, the head ‘flies’ over the disk governed by principles of gas bearing lubrication at high Knudsen number (mean free path/film thickness).

Talke (1995) concluded that the contact mechanics involves the friction and resultant wear at the head-disk interface created by repetitive stopping and starting, when surfaces touch. Protecting the magnetic layers then becomes of vital importance.

Gulino et al. (1986) looked at another consideration contributing to degradation during the cyclic starts and stops which involves the adverse effects of temperature rise at the interface of the head and disk. The random occurrence of surface roughness creates other challenges when addressing the contact and tribological problems. Accurately detailing the surface roughness of the head/disk interface is subject to instrument limitations such as resolution, as outlined by Oden et al. (1992).

2.2. Coatings

Protective coating layers offer promise of improved performance in many areas including injection molding, pharmaceutical and biomedical applications, military uses, and even sports and food service equipment to name a few. New coatings are being developed to withstand extreme temperatures for the purpose of providing non-stick surfaces in molds and dies, gears and bearings and military weapons. The temperature ranges in which some of these coatings are capable of performing are between 500°C in air to vacuum conditions of 1300°C. For adhesive performance at high temperatures, epoxy-based coatings for ceramics, glass, metals and plastics are being engineered to withstand −65°C to nearly 250°C. Thin film coatings of steel medical instruments and baking equipment as well as skis, are further examples of applications of protective coatings used in non-ideal conditions. In some instances coatings are not applied to the products directly, but rather used in the manufacturing process to become part of the base material. For products whose shapes prohibit easy coating, powder coatings are available that provide a layer of a Teflon-based resin preventing penetration from as small as pinhole size while providing a low-friction surface.

In many cases, coatings are modeled as a thin layer with different elastic properties than the substrate. Key geometric parameters are layer thickness, contact width and radius of curvature. The layer may have different slip and traction boundary conditions at the substrate interface. There are many possible combinations of such conditions and parameters, which have led to numerous classical papers as described in K.L. Johnson's text. The case where the layer thickness is much less than the contact width is particularly useful and can be described by elementary solutions in which the compressive normal stress is constant across the layer. However, the behavior is quite different for incompressible materials (Poisson's ratio $\nu = 0.5$). An interesting problem is that of a receding contact in which surfaces initially touch appreciably but then separate reducing the contact area due to the applied force. Reducing the contact area with force is the opposite tendency than usual contact mechanics. Among many such papers, a case is considered by Keer et al. (1972).

As described by Erdimer (1992), ceramic coatings have contributed to an increase in fatigue life in rolling element bearings due to the ceramic hardness and resistance to wear. For many revolutions often no noticeable wear is observed, however, at some critical point wear debris is present. Such accumulation of wear debris indicates the onset of fatigue failure of the bearing. In this study of TiN as a coating applied to base steels, fatigue life varied depending on the thickness of the coating film. A thin film TiN coating prolonged fatigue life while a thicker one resulted in fracture and delamination of the ceramic, consequently shortening the fatigue life of the base steel. Delamination results from inclusions, which serve as sites for crack nucleation. The subsurface shearing results in the formation of long grains running parallel to the sliding surface. Upon sliding, cracks run more easily along the grain boundaries rather than across them. Cracks grow through fatigue mechanisms and eventually coalesce into longer cracks parallel to the sliding surface and then link to the surface forming plate-like debris.

2.3. Gears and cams

Predicting service life of mechanical components is vital to providing cost-efficient designs of specific system components such as gears and cams. Often it is the case with gears that fatigue failure as a result of propagating cracks may unexpectedly and abruptly terminate their usefulness.

Blake and Draper (1994) describe the phenomenon of pitting, which commonly occurs on gears, as the result of surface material being removed due to cyclic loading causing small cracks to form which eventually grow in size until pieces of the material separate from the surface. Gear contacts involve both rolling and sliding mechanisms and the contact itself is nonconformal. As described above, the phenomenon of fatigue greatly influences gear life. However, for some materials, even under loads that are not severe, wear becomes the central contributing factor of service life. Monitoring gear wear during operation has been accomplished for different composite and polymer gear pairs and gears are found to fail from a variety of modes from excessive wear to tooth fracture.

Hooke et al. (1993) found that for gears with contacting surfaces made of polymers, high-energy dissipation could result under low loads. This heat generation may contribute to wear and eventual degradation of gears to failure. Studies show that for some materials, a transition from low to high wear rates is a result of increases in body and surface temperatures. Failure occurs when the material locally reaches its melting point.

Horng et al. (1994) experimentally observed wear and friction as a function of surface roughness in a gear-cam adapter used to simulate line contact. Here the main contributors to the increase in the rate of wear were found to be increases in asperity height and applied load with a decrease in rotational speed. When the surface roughness pattern was varied, the maximum rate of wear was found to occur for transversely oriented patterns versus longitudinal and oblique patterns.

2.4. *Biomechanical*

Artificial joint replacements have long been used to return function to degraded human joints due to arthritic and traumatic causes. One of the major concerns with prosthetic joints is the high volume of wear debris generated upon implantation. The tribological nature of implanted joints is that they are not fully lubricated as in healthy human joints and so consequently friction and wear are of considerable concern. Wear does occur in prostheses such as artificial hip and knee joints implying that there is at least in part, contact of the articulating surfaces. The material combinations greatly impact the amounts of wear particles formed. Countersurfaces are primarily those combinations of metal-on-polymer, metal-on-metal and ceramic-on-polymer. Stress conditions have been examined, for instance, in the use of ultra high molecular weight polyethylene as the polymer.

Barbour et al. (1997) found the wear rate not to be a function of magnitude of the applied load but rather the magnitude of the contact stress and the conditions under which the stress is applied, either steady or cyclic. Here, the effect of the steady stresses far surpass the cyclic stresses in minimizing wear generation.

To this end, soft layer joints are presently being considered by Unsworth (1995), because of the endurance of these types of joints under very high fatigue loads coupled with an increase in lubricating action.

2.5. *Seals*

Various operating conditions require a variety of seals to satisfactorily meet specific leakage requirements. For non-contacting seals, friction and the resultant wear is not applicable. However, mechanical seals designed for the purpose of contacting with an intended surface offer much lower leakage than their non-contacting counterparts. The design must adequately address the issues of friction and wear of the seal faces in order to obtain sufficient service life. Analyses of damped and undamped mechanical face seals as well as new advances in three-dimensional modeling of brush seals for new technology applications are burgeoning areas of interest.

In the study of damping effects on separation speed of contacting mechanical face seals, Green and Bair (1991), an analytical solution has been developed. Wear in the axial and angular modes, as well as separation speed, were found to be adversely affected by the support damping. However, wear was determined to be minimum at some contact force that produced an optimum operation speed below the separation speed.

New areas of seal technology include the brush seal and its use in controlling leakage flow in gas turbine engines. The contact mechanics involve the bristles against the rotating surface, emphasizing the importance for accurate wear studies to assess the loads and dissipated energy. Due to the complicated nature of the interaction between individual bristles of brush seals, Aksit and Tichy (1998) have conducted experimental tests and computer analyses. They seek to accurately investigate the central concerns: friction and wear characteristics between the bristles and rotor, frictional interactions between the bristles themselves, pressure loads and bristle stiffening, and the effects of temperature and cooling on wear. Each of these factors separately and in combination are vital for minimizing the wear rate of brush seals.

2.6. *Micro electo mechanical systems (MEMS)*

MEMS are small micrometer-scale devices, which serve as sensors, actuators, simple machines, etc. Tribology is expected to play a strong role in enabling this technology because forces on surfaces dominate body forces. At present, there are relatively few economically viable applications and most interest has focused on the fabrication process. Surface chemistry, adhesion and capillarity, and materials science have comprised most of the tribological interest, with avoidance of surface sticking and component life being the technological issues of prime importance. The minute scales create unique problems. Surfaces often stick permanently if they touch and the device has thus failed. A bearing surface may have a life of 10^6 cycles, however, subject to oscillation speed of 10^5 Hz, a useful life of only a few seconds may occur. A recent conference proceeding on MEMS describes many tribology issues, Bhushan (1998).

As to applications of solid mechanics in the tribology of MEMS, most appear to center around interactions between the elastic, capillary, and adhesive forces. The force between individual atoms and molecules, often described through a surface energy is referred to as the adhesive force. To this end, Greenwood (1997) describes adhesion of elastic spheres. Mastrangelo and Hsu (1993) discuss the interplay of the elastic and capillary forces, including an elasto-capillary number.

3. Research topics

Friction and wear occur in problems of a tribological nature and are intertwined with solid mechanics from investigations of contacting surface geometry. Global friction and wear are functions of the developed stress from asperity contacts, and the stresses leading to deformation of the asperities, either elastic or plastic. Material characteristics, such as shear strength at contacting asperities in turn effect the degree of asperity deformation.

The conditions from which friction and wear originate, such as sliding and rolling contacts, are present either individually or in combination. These motions give rise to tractions at the interface for various types of contact, from point or line to circular or elliptical. The traction components, normal traction or contact pressure and tangential traction due to friction are distributed over the contact area.

3.1. *Rough surface contacts*

Contact pressures and asperity interactions provide useful information when evaluating rough surface contacts. Computer models, often based on statistical assumptions, give methods for determining these quantities when it is necessary to understand the details of entire surfaces.

Stanley and Kato (1997) provide a numerical simulation which predicts contact pressures and surface normal displacements for a pair of nominally flat, parallel rough surfaces in two and three dimensions, making it particularly useful for characterizing surface topography. In addition to contact pressures, the effects of roughness on the distribution of real area of contact and the gaps between contacts have been considered.

In a recent contact model, Poon and Sayles (1994) numerically determine the load-area relationship in elastic-plastic contacts, providing a means to analyze rough surface profiles.

Earlier, a similar model was developed, Lee and Cheng (1992), for contacts between longitudinally oriented rough surfaces. The same work generated analytical relationships between the real area of contact and load and the average load and average gap between contacts.

Chang et al. (1988) calculated the frictional force at the contacts in metallic rough surfaces, taking into account surface forces and pre-stress in the contact area. Material properties and surface topography play important roles in the value of the coefficient of friction. The external force was found to have an effect on the static friction coefficient.

3.2. Cracks, fracture and fatigue

Murakami et al. (1994) have shown rolling and sliding contact under cyclic loading to contribute to pit formation. Contact pressure and frictional force factor significantly, with pitting occurring first due to shearing and subsequently in the tensile mode.

By intentionally introducing defects, Cheng et al. (1993) showed through a series of experimental and numerical analyses of rolling-sliding contact fatigue tests that the deformed geometry and friction coefficient contributed significantly to the maximum Mises stress. Surface crack initiation was also largely dependent on the contact pressure.

The phenomenon of fretting fatigue, where components fail in response to crack propagation is another example of the combination of tribology and contact mechanics. Fretting may be described as the slip that occurs repetitively to clamped components undergoing vibration. Although no global motion is taking place, micromotion in the form of slip in some areas and stick in others dominates, with small wear particles breaking loose from the surfaces.

Farris (1992) found that crack initiation from the bulk compression is present when the materials are dissimilar yet this same stress concentration is not evident in materials of the same elastic constants. Fretting fatigue life then may be predicted taking into account the applied bulk stress and local stress concentrations.

4. Conclusion

Although the term tribology is new, it is an ancient field with roots in antiquity. With every new mechanical technology a new set of tribological issues seems to arise, the friction and wear of small-scale devices like MEMS being a good example. Furthermore, mundane issues such as improvements in reliability, vibration control, and condition monitoring, require that strides continue to be made on the tribology of traditional machinery. The contributions made by solid mechanics to the field of tribology have brought advancements and improvements in many of these old and new areas. The impact of the combined aspects of tribology reaches into many areas of engineering and science, resulting in achievements and new developments of theoretical understanding and practical benefit. Looking to the next century, investigations of a tribological nature will continue to be necessary, as the mechanisms of machinery and mechanical systems become more complex.

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